12.1 Inscribed Polygons
12.2 Arc Length
Pre-AP Geometry

Name Key
Period ______________ Date ________

Section I: Inscribed Polygons

Figure I

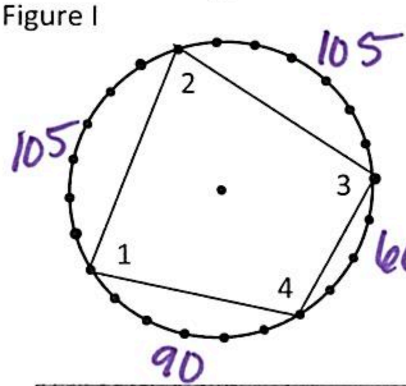
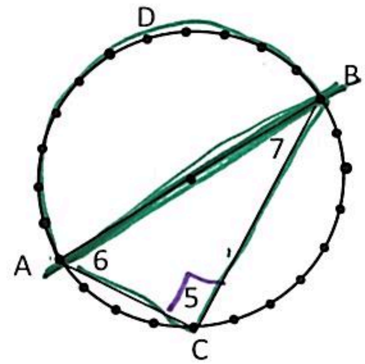


Figure II



	Measure of each angle		Measure of each angle	To find the measure of the intercepted arcs, count the dots
Figure I	m∠1 = 82.5	Figure II	m∠5 = 90°	m $\widehat{ADB}$ = 180
Figure I	m∠2 = 75	Figure II	m∠6 = 60	m $\widehat{BC}$ = 120
Figure I	m∠3 = 97.5	Figure II	m∠7 = 30	m $\widehat{AC}$ 60
Figure I	m∠4 = 105			

Conjecture:

1. If a quadrilateral is inscribed in a circle then the opposite angles are…..

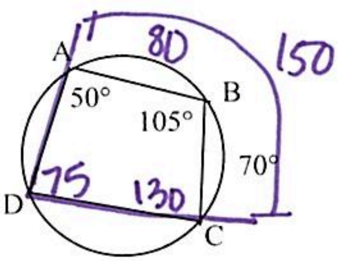
   Supplementary

2. If a right triangle is inscribed in a circle then the hypotenuse is…..

   the Diameter

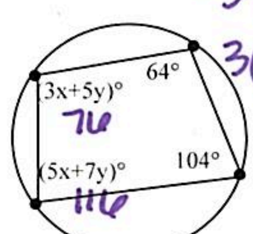
Practice:

1. Find the $m\widehat{AB}$



$m\widehat{AB} = 80$

2. Find x and y.



$-5(3x+5y=76) \rightarrow -\cancel{15x}-25y=-380$

$3(5x+7y=116) \rightarrow \cancel{15x}+21y=348$

$-4y=-32$

$\boxed{y=8}$

$3x+5(8)=76$

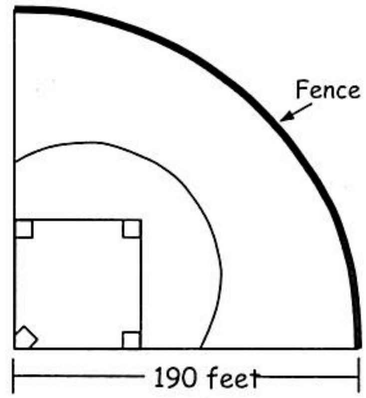
$3x+40=76$

$3x=36$

$\boxed{x=12}$

Section II: Arc Length (Textbook pages #1060-1064)

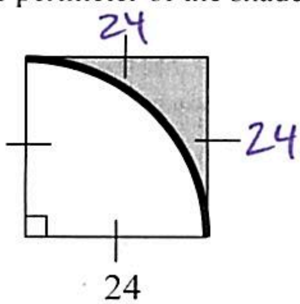
1) The high school athletic department needs to build the fence along the outfield of the softball field. What is the length of the outfield fence?



$\frac{1}{4}(2\pi\, 190)$

$\frac{1}{4}(380\pi) = \boxed{95\pi}$

2) Find the perimeter of the shaded region.



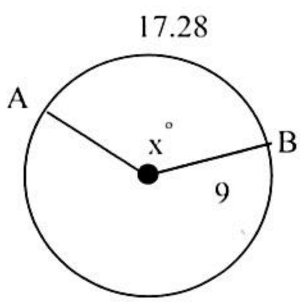
$\frac{1}{4}(2\pi \cdot 24)$

$= \frac{1}{4}(48\pi)$

$= 12\pi$

$\boxed{48 + 12\pi}$

3) Given the length of $\widehat{AB}$, find the measure of the central angle.

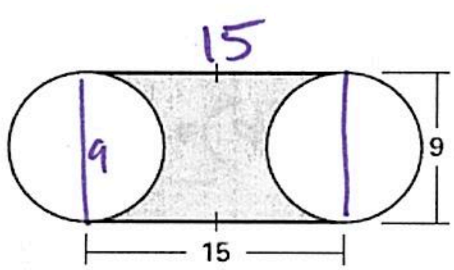


$\frac{x}{360} \cdot 2\pi 9 = 17.28$

$\frac{x}{360} \cdot 18\pi = 17.28$ $\boxed{x = 110^\circ}$

$\frac{x(18\pi)}{(18\pi)} = \frac{6{,}220.8}{(18\pi)}$

4) Find the perimeter of the shaded region.



$C = 9\pi$

$\boxed{P = 30 + 9\pi}$

Gears

Arc Length

12.2

LEARNING GOALS

In this lesson, you will:

- Distinguish between arc measure and arc length.
- Use a formula to solve for arc length in degree measures.
- Distinguish between degree measure and radian measure.
- Use a formula to solve for arc length in radian measures.

KEY TERMS

- arc length
- radian

Gears are used in many mechanical devices to provide torque, or the force that causes rotation. For instance, an electric screwdriver contains gears. The motor of an electric screwdriver can make the spinning components spin very fast, but the gears are needed to provide the force to push a screw into place. Gears can be very large or very small, depending on their application.

① Front of WS (12.1)

② Book pg. 1060 - 1064

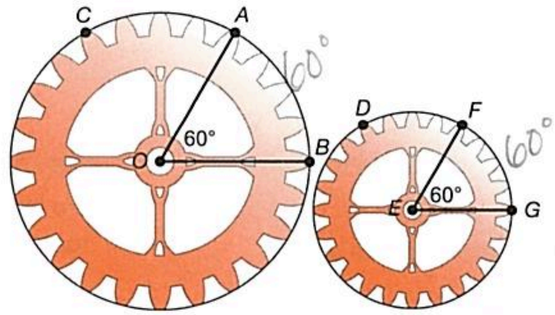
③ Finish back of WS

PROBLEM 1 Large and Small Gears

on their own

1. Consider the large gear represented by circle O, containing a central angle; $\angle AOB$, whose measure is equal to 60°; a minor arc, $\widehat{AB}$; and a major arc, $\widehat{ACB}$, as shown.

   Consider the small gear represented by circle E, containing a central angle; $\angle FEG$, whose measure is equal to 60°; a minor arc, $\widehat{FG}$; and a major arc, $\widehat{FDG}$, as shown.



   a. Is the large gear similar to the small gear? Explain your reasoning.

   yes, all circles are similar

   b. Is the length of the radii in the large gear proportional to the length of the radii in the small gear? Explain your reasoning.

   Yes all circles are similar

   c. Determine the degree measure of the minor arc in each circle.

   60°

   d. What is the ratio of the degree measure of the minor arc to the degree measure of the entire circle for each of the two gears?

   $$\frac{60°}{360°} = \frac{1}{6}$$

   e. The degree measure of the intercepted arc in the large gear is equal to the degree measure of the intercepted arc in the small gear, but do the two intercepted arcs appear to be the same length?

   NO one looks bigger

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2. Explain why Casey is incorrect.

Casey

The two minor arcs, $\overset{\frown}{AB}$ and $\overset{\frown}{FG}$, on the gears have the same measure, which is 60°. So, the two arcs are the same length.

Arc length is a portion of the circumference of a circle. The *length* of an arc is different from the *degree measure* of the arc. Arcs are measured in degrees whereas arc lengths are linear measurements.

To determine the arc length of the minor arc, you need to work with the circumference of the circle, which requires knowing the radius of the circle.

As class

3. If the length of the radius of the large gear, or line segment OB is equal to 4 centimeters, determine the circumference of circle O.

$C = 2\pi r = 2\pi(r) = 8\pi$

4. Use the circumference of circle O determined in Question 3 and the ratio determined in Question 1, part (d) to solve for the length of the minor arc.

$\frac{1}{6}(8\pi) = \boxed{\frac{4}{3}\pi}$

5. If the length of the radius of the small gear, or line segment EF, is equal to 2 centimeters, determine the circumference of circle E.

$C = 2\pi(2) = 4\pi$

6. Use the circumference of circle E determined in Question 5 and the ratio determined in Question 1, part (d) to solve for the length of the minor arc.

$\frac{1}{6}(4\pi) = \boxed{\frac{2}{3}\pi}$

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To determine arc length, s, you multiplied the circumference of the circle by a fraction that represents the portion of the circumference determined by the central angle measure, m. There is a proportional relationship between the measure of an arc length of a circle, s, and the circumference of the circle.

$$s = \frac{m}{360^\circ} \cdot \text{circumference}$$

$$\frac{s}{\text{circumference}} = \frac{m}{360^\circ}$$

$$\boxed{\text{Arc Length} = \frac{x}{360} \cdot 2\pi r}$$

fraction

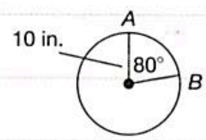
C.

The formula for arc length can also be written as follows.

$$s = \frac{m}{360^\circ} \cdot \text{circumference}$$

$$s = \frac{m}{360^\circ} \cdot 2\pi r$$

You can apply the formula $s = \frac{m}{360^\circ} \cdot 2\pi r$ to determine the measure of an arc for a circle with a radius of 10 inches and central angle of 80°.



Ex.) $\frac{80}{360} \cdot 2\pi(10)$

$\frac{2}{9} \cdot 20\pi$

$\boxed{\frac{40\pi}{9}}$

$$s = \frac{m}{360^\circ} \cdot 2\pi r$$

$$m\widehat{AB} = \frac{80^\circ}{360^\circ} \cdot 2\pi(10)$$

$$m\widehat{AB} = \frac{2}{9} \cdot 20\pi$$

$$m\widehat{AB} = \frac{40}{9}\pi$$

The measure of arc length AB is $\frac{40}{9}\pi$, or approximately 14 inches.

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The formula implies that the ratio of the arc length to the radius, $\frac{s}{r}$, is directly proportional to m, the measure of the central angle.

$$s = \frac{m}{360^\circ} \cdot 2\pi r$$

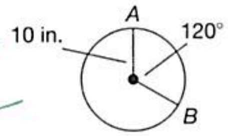
$$s = \frac{m}{180^\circ} \cdot \pi r$$

$$\frac{s}{r} = \frac{\pi}{180^\circ} \cdot m$$

7. Apply the proportional relationship between the measure of an arc length of a circle and the circumference of a circle to calculate the arc length of each circle. Express your answer in terms of π.

a.

Together

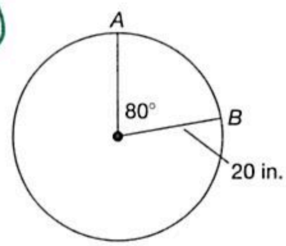


$$\frac{120}{360} \cdot 2\pi(10)$$

$$\frac{1}{3} \cdot 20\pi = \boxed{\frac{20}{3}\pi}$$

$$\boxed{20.944}$$

b.



$$\frac{80}{360} \cdot 2\pi(20)$$

$$\frac{2}{9} \cdot 40\pi = \boxed{\frac{80\pi}{9}}$$

$$\boxed{27.925}$$

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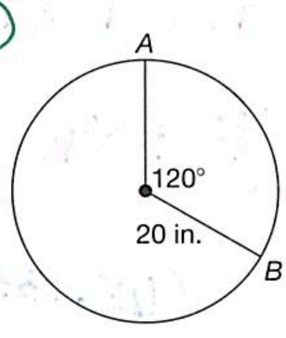
c.



$$\frac{120}{360} \cdot 2\pi(20)$$

$$\frac{1}{3} \cdot 40\pi = \boxed{\frac{40\pi}{3}}$$

$$\boxed{41.888}$$

8. Look at Question 7, parts (a) and (c) as well as Question 7, parts (b) and (d). In each pair, the central angle is the same but the radius has been doubled. What effect does doubling the radius have on the length of the arc? Justify why this relationship exist?

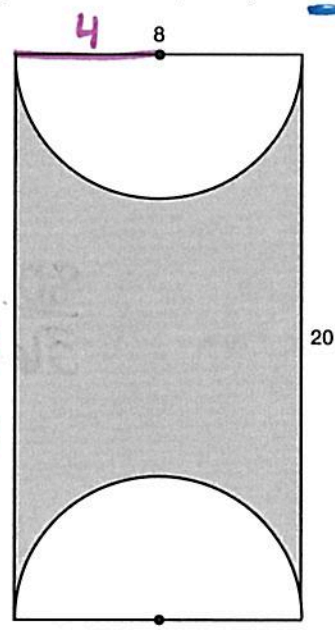
Skip

Doubles

Challenge!

9. Two semicircular cuts were taken from the rectangular region shown. Determine the perimeter of the shaded region. Do not express your answer in terms of π.

exact!



20

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$C = 2\pi(4)$

$C = 8\pi$

$20 + 20 + 8\pi$

$\boxed{40 + 8\pi}$

(65.133)